SOLUTION – DETERMINANTS

Q.1. Solve [Any 3] (2 Marks each) **(06)**

1) Given $\begin{vmatrix} x & 1 & 3 \\ 3 & 1 & x \\ 2 & 1 & 3 \end{vmatrix} = 0$,

$\therefore\ x\begin{vmatrix} 1 & x \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 3 & x \\ 2 & 3 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = 0$

$\therefore$ x (3 – x) – 1(9 – 2x) + 3(3 – 2) = 0

$\therefore$ $3x - x^2 - 9 + 2x + 3 = 0$

$\therefore$ $-x^2 + 5x - 6 = 0$

$\therefore$ $x^2 - 5x + 6 = 0$

$\therefore$ (x – 3) (x – 2) = 0

$\therefore$ x – 3 = 0 or x – 2 = 0

$\therefore$ x = 3 or x = 2

2) Let D = $\begin{vmatrix} 0 & y & z \\ -y & 0 & x \\ -z & -x & 0 \end{vmatrix}$

Taking -1 common from R_1, R_2 and R_3 each

$= (-1)^3 \begin{vmatrix} 0 & -y & -z \\ y & 0 & -x \\ z & x & 0 \end{vmatrix}$

$= -\begin{vmatrix} 0 & -y & -z \\ y & 0 & -x \\ z & x & 0 \end{vmatrix}$

Interchange rows and columns

$= -\begin{vmatrix} 0 & y & z \\ -y & 0 & x \\ -z & -x & 0 \end{vmatrix}$

$\therefore$ D = -D

$\therefore$ D + D = 0

$\therefore$ 2 D = 0

$\therefore$ D = 0

3) $\begin{vmatrix} 1 & 1 & 1 \\ 10 & 11 & 12 \\ 100 & 101 & 102 \end{vmatrix}$

$= 1\begin{vmatrix} 11 & 12 \\ 101 & 102 \end{vmatrix} - 1\begin{vmatrix} 10 & 12 \\ 100 & 102 \end{vmatrix} + 1\begin{vmatrix} 10 & 11 \\ 100 & 101 \end{vmatrix}$

$\therefore$ (1122 – 1212) – (1020 – 1200) + (1010 – 1100) = 0

$\therefore$ - 90 + 180 - 90 = 0

4) $\begin{vmatrix} 1 & 2 & 3 \\ 12 & 13 & 14 \\ 33 & 34 & 35 \end{vmatrix}$

$\therefore\ a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= 1x \begin{vmatrix} 13 & 14 \\ 34 & 35 \end{vmatrix} - 2x \begin{vmatrix} 12 & 14 \\ 33 & 35 \end{vmatrix} + 3 \begin{vmatrix} 12 & 13 \\ 33 & 34 \end{vmatrix}$

$= 1 \text{ x } (455 - 476) - 2(420 - 462) + 3(408 - 429)$

$= -21 + 84 - 63$

$= 0$

Q.2. Solve [Any 4] (3 Marks each) (12)

1) LHS = $\begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

Taking 'a' common from C_1, 'b' from C_2 'c' from C_3

$= abc \begin{vmatrix} x & y & z \\ a & b & c \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \end{vmatrix}$

$= \begin{vmatrix} x & y & z \\ a & b & c \\ \frac{abc}{a} & \frac{abc}{b} & \frac{abc}{c} \end{vmatrix}$

$= \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

= R.H.S.

2) The given equations are consistent, if

$\begin{vmatrix} 5 & 6 & 17 \\ 2 & 3 & 8 \\ 1 & 1 & 3 \end{vmatrix} = 0$, provided $\begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix} \neq 0$

Now,

LHS $= 5(9 - 8) - 6(6 - 8) + 17(2 - 3)$

$= 5(1) - 6(-2) + 17(-1)$

$= 5 + 12 - 17 = 17 - 17 =$ RHS

$\therefore$ the given equations are consistent.

3) The given equations are consistent.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} k+1 & k-1 & k-1 \\ k-1 & k+1 & k-1 \\ k-1 & k-1 & k+1 \end{vmatrix} = 0$$

$R_1 \to 2R_1 - \frac{1}{2}(R_2 + R_3)$

$$\begin{vmatrix} 2 & -1 & -1 \\ k-1 & k+1 & k-1 \\ k-1 & k-1 & k+1 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$

$$\begin{vmatrix} 3 & 0 & -1 \\ 0 & 2 & k-1 \\ -2 & -2 & k+1 \end{vmatrix} = 0$$

$\therefore$ 3(2k + 2 + 2k - 2) – 0 - 1 (0 + 4) = 0

$\therefore$ 12k - 4 = 0

$\therefore k = \frac{1}{3}$

4) Put $\frac{1}{x-1} = a$, $\frac{1}{y+1} = b$, $\frac{1}{z} = c$

$\therefore$ the given equations become

2a – b + 2c = 1

3a + 2b - c = 1

a - 3b - 3c = 2

$$\therefore D = \begin{vmatrix} 2 & -1 & 2 \\ 3 & 2 & -1 \\ 1 & -3 & -3 \end{vmatrix}$$

= 2(-6 – 3) + 1(-9 + 1) + 2(-9 – 2)

= - 18 – 8 – 22 = - 48 $\neq$ 0

$$D_a = \begin{vmatrix} 1 & -1 & 2 \\ 1 & 2 & -1 \\ 2 & -3 & -3 \end{vmatrix}$$

= 1(-6 – 3) + 1(-3 + 2) + 2(-3 – 4)

= - 9 – 1 – 14 = - 24

$$D_b = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 1 & -1 \\ 1 & 2 & -3 \end{vmatrix}$$

= 2(-3 + 2) - 1(-9 + 1) + 2(6 – 1)

= - 2 + 8 + 10 = 16

$$D_c = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

= 2(4 + 3) + 1(6 - 1) + 1(-9 – 2)

= 14 + 5 - 11 = 8

$\therefore$ By cramer's rule,

$a = \frac{D_a}{D} = \frac{-24}{-48} = \frac{1}{2}$

$b = \frac{D_b}{D} = \frac{16}{-48} = -\frac{1}{3}$

$c = \frac{D_c}{D} = \frac{8}{-48} = -\frac{1}{6}$

$\therefore\ a = \frac{1}{x-1} = \frac{1}{2},$

$\therefore\ b = \frac{1}{y+1} = -\frac{1}{3},$

$\therefore\ c = \frac{1}{z} = -\frac{1}{6}$

$\therefore$ x – 1 = 2, y + 1 = -3, z = -6

$\therefore$ x = 3, y = -4, z = -6.

5) LHS = $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

By $R_2 - R_1$ and $R_3 - R_1$, we get,

LHS = $\begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-bc \end{vmatrix}$

$= \begin{vmatrix} 1 & a & bc \\ 0 & -(a-b) & c(a-b) \\ 0 & c-a & -b(c-a) \end{vmatrix}$

By taking (a – b) common from R_2 and (c - a) common from R_3, we get,

LHS $= (a-b)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & 1 & -b \end{vmatrix}$

= (a – b) (c – a) [1(b - c) – a(0 - 0) + bc(0 - 0)]

= (a – b) (c – a) (b - c)

= (a – b) (b - c) (c – a)

= RHS

6) consider $\begin{vmatrix} 1 & 2x & 4x^2 \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 0$

We know that the value of a determinant is zero, if its two rows are identical.

$\therefore$ by taking $R_1 \equiv R_2$, we get,

2x = 4 and $4x^2 = 16$, i.e., $x^2 = 4$

$\therefore$ x = 2

Also, by taking $R_1 \equiv R_3$, we get,

$2x = 1$ and $4x^2 = 1$, i.e., $x^2 = \frac{1}{4}$

$\therefore x = \frac{1}{2}$

Hence, $x = 2$ or $x = \frac{1}{2}$

Q.3. Solve [Any 3] (4 Marks each) (12)

1) $LHS = \begin{vmatrix} b+c & c & b \\ c & c+a & a \\ b & a & a+b \end{vmatrix}$

By $C_1 + (C_2 + C_3)$, we get,

$LHS = \begin{vmatrix} 2(b+c) & c & b \\ 2(c+a) & c+a & a \\ 2(a+b) & a & a+b \end{vmatrix}$

By taking 2 common from C_1, we get,

$LHS = 2\begin{vmatrix} b+c & c & b \\ c+a & c+a & a \\ a+b & a & a+b \end{vmatrix}$

By $C_1 - (C_2 + C_3)$, we get,

$LHS = 2\begin{vmatrix} 0 & c & b \\ -a & c+a & a \\ -a & a & a+b \end{vmatrix}$

By $C_2 + C_1$ and $C_3 + C_1$, we get,

$LHS = 2\begin{vmatrix} 0 & c & b \\ -a & c & 0 \\ -a & 0 & b \end{vmatrix}$

By taking a, c, b common from C_1, C_2, C_3 respectively,

$LHS = 2(acb)\begin{vmatrix} 0 & 1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$

$= 2abc\,[0(1 - 0) - 1(-1 + 0) + 1(0 + 0)]$
$= 2abc(0+1+1)$
$= 2abc\,(2)$
$= 4abc$
$= RHS$

2) The equations of the given lines are
$3x + y = 2$
$11x + 2y = 3$

2x – y = - 3

If the lines are concurrent then

$$\begin{vmatrix} 3 & 1 & 2 \\ 11 & 2 & 3 \\ 2 & -1 & -3 \end{vmatrix} = 0$$

$$\text{LHS} = \begin{vmatrix} 3 & 1 & 2 \\ 11 & 2 & 3 \\ 2 & -1 & -3 \end{vmatrix}$$

= 3(-6 + 3) - 1(- 33 – 6) + 2 (-11 - 4)

= - 9 + 39 – 30 = 0 = RHS

Hence, the given lines are concurrent.

3) We write the given equation in the form

px + qy = r as;

ax + by = -c, cx + ay = -b, bx + cy = -a

since these equations are consistent,

$$\begin{vmatrix} a & b & -c \\ c & a & -b \\ b & c & -a \end{vmatrix} = 0$$

$\therefore$ a(- a^2 + bc) – b(-ac + b^2) – c(c^2 - ab) = 0

$\therefore$ - a^3 + abc + abc – b^3 – c^3 + abc = 0

$\therefore$ $a^3 + b^3 + c^3 = 3abc$

4) Let A $\equiv$ (a, b), B $\equiv$ (c, d) and C $\equiv$ (a – c, b – d).

Since points A, B, C are collinear,

Area of Δ ABC = 0

$$\therefore \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & a & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

$\therefore$ a(d - b + d) – b(c – a + c) + 1(bc – cd – ad + cd) = 0

$\therefore$ a(2d - b) – b (2c - a) + (bc – ad) = 0

$\therefore$ 2ad - ab – 2bc + ab + bc – ad = 0

$\therefore$ ab – bc = 0

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